Numerical Methods for the Model of Spatial Random Permutations and a Connection to Schramm-Loewner Evolution II

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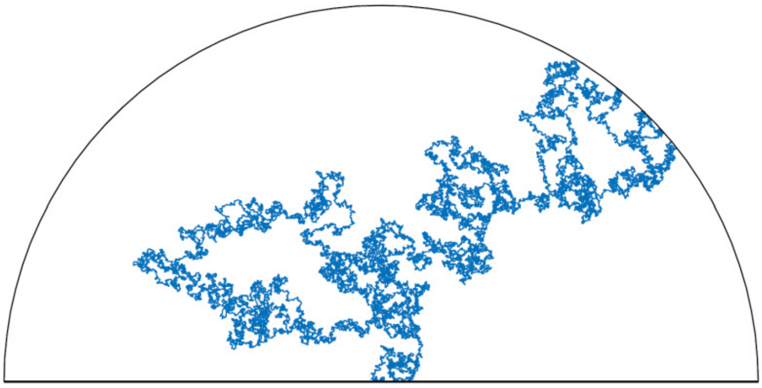

Figure 1: A typical path in the SRP model on a lattice with radial boundary and Radius 500. The path starts in (0,0) and the endpoint was fixed on the boundary at point (313,389). The parameter of the model was alpha=0.5.

Introduction

We made several numerical calculations regarding the model of Spatial Random Permutations (SRP). This is a model in statistical mechanics and is of recent interest. In this model we look at random permutations of lattice points, which can also be identified as a directed Graph, where each point is connected with its target point via an arrow, such that cycles in the permutation form loops in the graph. As typical in models in statistical mechanics the probability of a certain realization depends on the energy of the system, where the energy and probability of a realization are defined in such a way that short arrows are more likely. The degree to which long arrows get penalized depends on the parameter alpha of the system. We modify this model in the way that we force a path in the system which starts at the origin and goes to a boundary point. We believe that there is a connection between this model and the concept of Schramm-Loewner Evolution (SLE), which is a certain family of random curves in the complex half plane. These curves can also be characterized by a real valued function, the so-called driving function. In the case of SLE this function is a Brownian Motion. The SRP model is hard to handle analytically, so we want to show some connections via numerical calculations. Because we want to look at geometrical quantities and therefor have to look at large systems to avoid finite effects, we need a lot of computation power. In the previous period the main quantity we looked at is the probability for a path in the SRP model to pass left to a certain point in a reasonable sense. In the case of SLE this probability is given by Cardy´s formula. In this period, we wanted to hint the connection directly by using an algorithm which approximates the driving function of a given curve. Because the Schramm-Loewner theory works only for curves that does not intersect themselves and curves in SRP usually does, we need some way to make the curves crossfree.

Methods

We use Markov Chain Monte Carlo methods, more precisely the Metropolis Chain method, to get samples of our system and compute several quantities with that. In that sense we start with a permutation and mix it in several steps by changing the targets of some points in each step. But we change only if either the new permutation has less energy or at random, where the probability in that case depends on the energy difference. We found out that in order to get reasonable independent samples a very high amount of mixing steps needs to be done. This is one reason why it takes high computation power to simulate this model. In this period, we made more analysis on the correlation between consecutive samples and found out that even more steps are needed. The code is written in C and we use MPI to parallelize the computation of the samples. To generate the random numbers, we use the Mersenne Twister (SFMT).

Results

We thought of multiple ways to get rid of the crosses in our samples. Therefor, we successfully established some algorithms that removes crosses from a path. For the new paths we used the algorithm that approximates the driving function and checked if these are Brownian Motions. For the test of Brownian Motion we had to deal with several issues, since it seems like the model should be adequate. Especially, one should only consider the path until a time when he is still far away from the border to avoid finite size affects. Additionally, the correlation between samples should be small so the amount of mixing steps should be large. As a result, we still need to make computations on a larger lattice. But the results we got yet suggest a connection.

Discussion

However, we need yet to do further calculations on larger lattices to get more precise results and we need to make more statistical analysis of the results. Also, we want to do calculations for other parameters of the system. Also, we did calculations for the original paths (with crosses) and they also hint a connection. This leads to the guess that there might be a useful theory also for curves which intersect themselves.

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