

# Numerical Methods for Viscoelastic Single- and Multiphase Flows

Researchers  
Matthias Niethammer, Dr.-Ing.  
Holger Marschall and Christian  
Kunkelmann

Principal Investigator  
Prof. Dr. Dieter Bothe

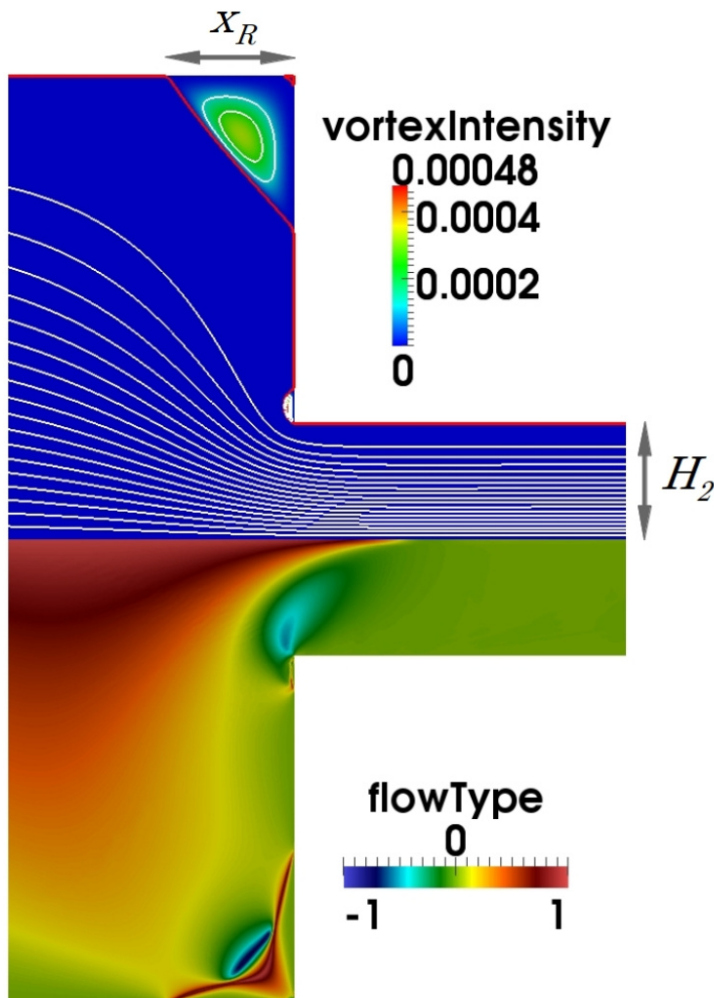
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## Introduction

The „High Weissenberg Number Problem“ (HWNP) has been a major challenge in computational rheology. It refers to the loss of convergence of all numerical methods beyond some limiting value of the fluid elasticity, quantified by a critical Weissenberg number. The critical value varies with the problem, in particular with regard to the flow geometry and the fluid constitutive model. Although a complete solution is not known until today, several effective numerical stabilization methods have been developed to cope with the HWNP.[1,2,3] However, there is a multitude of constitutive models describing viscoelastic material behavior, with which the stabilization approaches are to be combined. For the first time, this combination has been realized in a general way by deriving model-independent forms of the stabilized equations.

## Methods

We have developed a generic stabilization library that provides full combinatorial flexibility between different kinds of rheological models on the one hand, and distinct stabilization methods on the other hand. Our framework has been implemented by massive use of generic C++ template programming, runtime polymorphism and overloading. It is build on top of the open source library OpenFOAM, which provides a finite volume method on general unstructured meshes.

## Results

The planar entry flow in a 4:1 contraction is a benchmark problem in computational rheology. In Figure 1, we present results of an Oldroyd-B fluid in a 4:1 contraction. Additionally, some published values from the literature are plotted. Our purpose is to validate the results of two different stabilization methods from our new library, namely, the logarithm conformation tensor representation (LogC) and the squareroot conformation tensor representation (SqrtC). Our results for this benchmark test agree well with those from the literature. For standard numerical methods, such as the stress tensor representation (Str), the critical Weissenberg number is reported as  $Wi = 2.8$  [4]. In contrast, our stabilization framework has been found to be both robust and accurate for much higher Weissenberg numbers up to  $Wi = 100$  [5]. All simulations have been carried out using a finite-volume method on unstructured meshes. The computational mesh with a total number 146250 cells has been decomposed into 12 subdomains for reasons of parallelisation.

## Discussion

We are currently validating our numerical methods in several computational benchmark problems over a large range of Weissenberg numbers. Our purpose is to present mesh converged solutions on different computational meshes and to compare the numerical methods, particularly regarding accuracy

and robustness. Moreover, we are extending our code in order to simulate viscoelastic two-phase flows. The authors would like to acknowledge BASF SE for the financial support of this work, as well as Christian Kunkelmann, Erik Wassner and Sebastian Weisse for their cooperation, assistance, and enlightening discussions.

## Figures

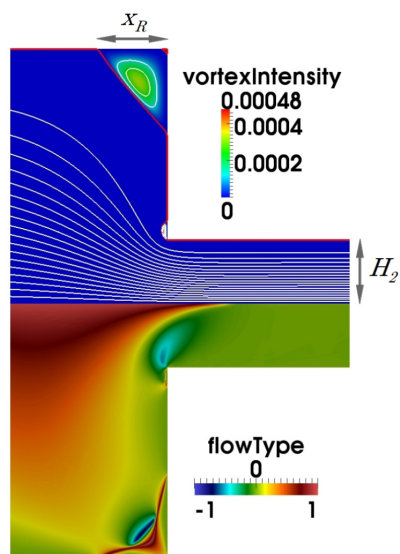


Fig. 1: Benchmark solutions of an Oldroyd-B fluid in a 4:1 contraction. Flow pattern at  $Wi=2$ .

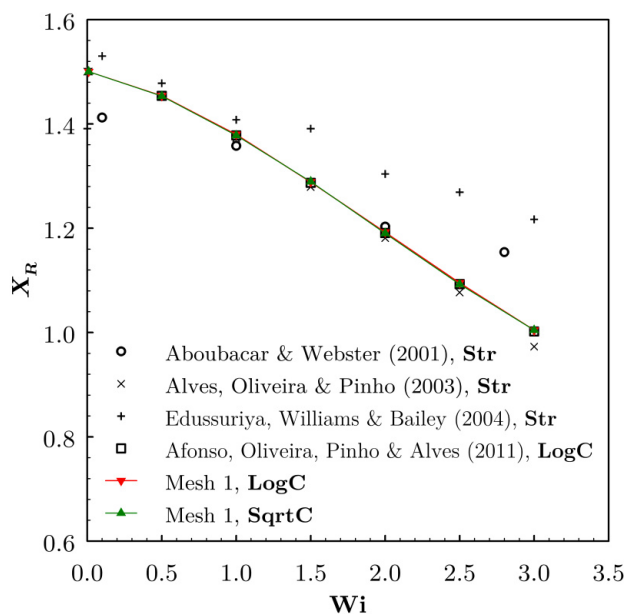


Fig. 2: Variation of the corner vortex size  $X_R = x_R / H_2$  with the Weissenberg number.

## Reference

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