Hessisches Kompetenzzentrum für Hochleistungsrechnen

Adaptive Numerical Wavelet Frame Methods for Elliptic and Parabolic Operator Equations, Inverse Problems, and Stochastic Evolution Equations

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Project Areas
Mathematics

Clusters
MaRC2 Cluster Marburg

Additional Software
C++

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Introduction

Many real-world phenomena are modelled by partial differential equations. Even in relatively low dimensions, the discretization of the equations usually leads to systems involving millions of unknowns. In such a situation, for the efficient numerical computation of a highly accurate approximate solution, adaptive schemes are often indispensable. In recent years, adaptive methods based on wavelets have been brought into focus. It is known that wavelets are predestined to resolve well local phenomena, such as singularities, while smooth data can be coded with very few coefficients. In different research projects, we have developed, implemented, and tested new adaptive wavelet algorithms for the solution of operator equations.

Methods

The starting point of our numerical methods is the construction of a proper wavelet collection spanning the solution space. In order not to be forced to cope with the often complicated construction of a wavelet Riesz basis on a general domain, we have focused on the weaker concept of wavelet frames [1, 2]. Based on this approach, in a new type of parallel adaptive wavelet frame methods has been introduced. These schemes have been successfully tested on the MaRC Linux Cluster placed at the Marburg University Computing Center. Splitting the domain into overlapping parametric images of streched cubes and treating each parts in parallel turned out to be asymptotically optimal. The methods have again been tested on MaRC.

Outlook

Adaptive methods have been successfully used for many practical problems. Nevertheless, it is our goal to further enhance the performance of the adaptive wavelet algorithms. One approach is to employ newly developed generalized tensor wavelets [3], since they give rise to dimension independent convergence rates. We are currently also investigating the wavelet version of hp-refinement schemes, the so-called adaptive quarklet methods [4, 5]. In the near future, the resulting adaptive algorithms will be implemented and tested on MaRC.

Reference

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